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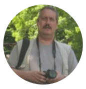
Gábor Molnár
Eötvös Loránd University



Gábor Timár
Eötvös Loránd University

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INVERSION APPLICATION IN CARTOGRAPHY: ESTIMATION OF THE PARAMETERS OF THE BEST FITTING CASSINI-PROJECTIONS OF THE FIRST HABSBURG MILITARY SURVEY

GÁBOR MOLNÁR[1]–GÁBOR TIMÁR[2]

[1]*MTA–ELTE Geological, Geophysical and Space Science Research Group*
molnar@sas.elte.hu
[2]*Department of Geophysics and Space Science*

Abstract. Map projection parameters and metadata (location of projection origin, pixel size of the scanned maps) are estimated using classical inversion-based estimation methods for the scanned survey sheets of the First Military Survey (end of 18th century) of the Habsburg Empire. The studied sheets cover five regions of the Empire: historical Hungary, Lower Austria, Bohemia, Croatia and Galicia. The sheets are organized into virtual mosaics according to these regions. For each region, the image coordinates in these mosaics and WGS84 geodetic coordinates from Google Earth are determined for several hundred identified points (ground control points). The map projection was modeled by the Cassini (Cassini-Soldner) projection and their projection equation set.

Using this approximation, the linear fit of the virtual mosaics to the Google Earth base layers was carried out with the same error magnitude (generally a few hundred meters, in extreme cases 2–3 kilometers), which was provided by the quadratic fit using modern coordinate systems. This implies that the Cassini projection is a good mathematical model of these map products. The longitude values of the projection center of the investigated regions are given and the latitude values of the projection centers proved to be arbitrary constants. In the case of Lower Austria, the resulting prime meridian is near the Stephansdom of Vienna, within the mentioned error margin, which indicates that this landmark might have been used for projection origin even in this early cartographic work.

1. INTRODUCTION

1.1. The First Military Survey of the Habsburg Empire

The First Military Survey of the Habsburg Empire was an outstanding geodetic and cartographic project of the second part of the 18^{th} century. Its result is a huge set of 1:28,800 scale manuscript maps, covering the actual territory of the Empire: Austria without its modern western territories, Bohemia, Galicia, historical Hungary with Croatia and Transylvania, as well as the Austrian Netherlands, more or less the present-day Belgium [1] [2] [3].

The survey was carried out on the order of Empress Maria Theresa, issued after the Seven Years War (1750–57). She and her headquarters realized that the Austrian armies needed a high-scale and precise map product for the territories of the empire. In cartographic history, the predecessor of this survey was the one of France, carried out by Cassini and Maraldi [4], slightly earlier than the Austrian work [5]. The scale of the French map system was 1:86,400, so the Austrian survey can be considered to be the first high-scale military topographic map system, and definitely the first one of the Central European region.

1.2. Georeference of the historical maps

Historical cartographic products have importance as archival documents, carrying information of the time of their creation. In addition, in many cases we can define mathematical relations between their contents and modern map grids. The result of this, a so-called geo-referred map, can be fit to any modern cartographic dataset or map, e.g. to the layers of Google Earth. This way, the original content of the old map can be analyzed also spatially, in any GIS (Geographic Information System) environment [6].

To make a geo-referred map of any old cartographic product, we have to define or estimate its original map projection, and in case of high-scale maps also the geodetic datum (for their detailed description, see [6]). Altogether, these data give the planar coordinate system of the map. Besides this, we have to provide an algorithm to give the coordinates in this system for all pixels of the scanned version of the map. In a GIS environment, a map rectified in a coordinate system defined by the map grid (projection) and the geodetic datum can be converted to any other cartographic coordinate system, especially to the native system of Google Earth.

The fit is regularly marred by some horizontal errors. The main error sources are: errors of the original geodetic survey; inaccurate grid and datum estimation by our process, and drawing errors/generalizations. In the case of the First Military Survey, the errors occurring after ordinary ground control point (GCP) rectification can be up to 1–2 kilometers [7].

1.3. The Cassini-projection as a possible mathematical model of the unknown map grid

According to the literature, there is an agreement that the First Military Survey had no geodetic basis [2]. However, the authors are convinced that the survey could not have been carried out, even with these high errors, without some geodetic background. It is known that there were baseline measurements at the time of the survey in Austria and also in Hungary [8]. Joseph Liesganig, who led these operations, was also the coordinator of the First Survey works in Galicia [9]. The determination of astronomical position, which is the very basis of any geodetic survey, was at quite a high level, even in determining the longitude differences between far prime meridians, e.g. those of Paris, Vienna, Greenwich and Ingolstadt, by special methods [10]. The lack of documentation of a geodetic survey in the Austrian archives is not evidence in itself. Even we could not accomplish this survey without some geodetic background, with the technology of the 18th century, with such relatively high global accuracy.

If we assume the existence of some (unknown) geodetic basis, an important question is what was the map projection of the resulting sheets. In short, the map projection is the function set between the geographical (latitude, longitude) and the map planar coordinates. According to the literature, the survey sheets have no projection, their system is 'projectionless'.

However, for a georeference method, we need at least some assumption of the map projection type. An obvious selection is the Cassini-projection [11], the method used for the map of France [12] [13]. We must emphasize that we do not state that the projection of the First Military Survey is exactly the Cassini-projection – we simply use this projection as a mathematical model of their native grid.

1.4. The goal of this study

In this study, we collect a set of ground control points (GCPs) in order to derive the optimum projection parameters – the parameter set that provides the minimum horizontal difference between the points' real and computed position in modern systems. Here the word 'computed' refers to the method of the georeference, coordinates computed from the object positions in the map grid, in practice in the scanned image matrix. Our study contains two important regions of the former Habsburg Empire: Lower Austria (with Vienna) and Hungary. The estimation of these projection parameters is only the first step in the accurate georeferencing of these survey sheets [14]; however it falls within the scope of this volume: in the field of inversion methods.

2. THE PROCESSED DATASET

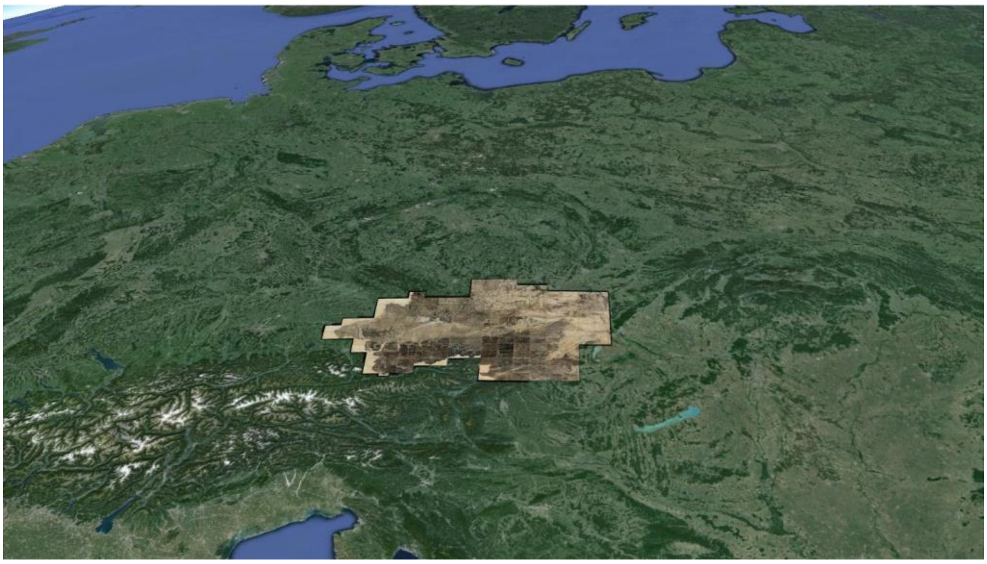

Figure 1
The mosaic of the Lower Austrian sheets of the First Military Survey
on the Google Earth geolayer: the target area of the present study

We used several hundred GCPs from the investigated regions, with an average density of 2–3 points per map sheet, or in other words, cca. one point per 100 km^2. For each point, the following data items are given:

- Geodetic coordinates in the WGS84 system, read from Google Earth, and
- Cartesian (planar) coordinates in the map mosaic grid.

To obtain the Cartesian coordinates, a virtual map mosaic was constructed for the map sheets belonging to each investigated region (Figure 1). Thus, the origins of these planar systems were taken at the upper left (north-western) corner of these mosaics.

We considered these planar coordinate systems as Cassini-type projected ones, with still unknown map projection parameters. The Cassini-projection has four compulsory parameters to be defined in any cartographic applications: the geodetic coordinates of the projection center in the base ellipsoid (a longitude and a latitude; two values) and the coordinates of the projection center in the projected plane (two length data in meters). Using this input data set, we planned to estimate the following parameters:

- the pixel size of the scanned map sheet images;
- the Cassini-coordinates of the origins (NW corner of the mosaics) and
- location of the map projection in the WGS84 system.

In this approach, the projection center's planar coordinates are set to zero in both directions and the mosaic is placed into this plane by the coordinates of the NW corner. With the above data, the virtual map mosaic can be geo-referred – the location of each pixel is given, thus the old map can be placed into a modern geodetic reference system, or, more practically, shown as a correctly placed overlay in the Google Earth environment, for popular publications.

3. METHOD: THE ESTIMATION OF THE PROJECTION PARAMETERS

To estimate the above-listed parameters, we shall start from the direct projection equations of the Cassini projection [11]:

$$\begin{aligned} x &= R \cdot \arcsin\left(\cos\Phi \cdot \sin\left(\lambda - \lambda_0\right)\right) \\ y &= R \cdot \left[\arctan\left[\tan\Phi / \cos\left(\lambda - \lambda_0\right)\right] - \Phi_0\right] \end{aligned} \tag{1}$$

where R is the radius of Earth, Φ and λ are latitude and longitude of the point, Φ_0 and λ_0 are the latitude and longitude of the center of the projection, x and y are the Cartesian coordinates of the projected point.

The usual approach of the inversion for parameter estimating is to provide a series of equations expressing the partial derivatives of Eq. (1), with respect to the parameters to be estimated ([15]; for more practical considerations cf. e.g. [16] [17] [18]). In the present case we could apply numerical derivatives instead of the analytical derivatives. However, in practice these parameters can be divided into two groups. According to the definition of the Cassini-projection, the only real independent parameter here is the longitude of the projection origin. The latitude of the projection origin and the south-northern location of the map plane together show total equivalency in the parameter estimation. Practically, this latitude value was set intentionally to around the mid-latitude of the investigated regions. All of the other parameters are expressed from the above ones, while the only independent parameter – the origin's longitude – was estimated by a single one-dimensional iteration.

4. Results

Our results are considered to be of two kinds:

- the accuracy of the fitting of the map mosaic to the modern coordinate systems, and
- the longitude values of the projection origins of some First Military Survey regions.

The accuracy is considerably worse than that of the Second Military Survey [19]. In our case, the maximum displacements are up to 2.4 kilometers for Lower Austria and even more (up to 3 kilometers) in historical Hungary. This implies that a further refinement of the geo-reference is still needed for more exact scientific analyses; we refer to this in the last section of this paper. The displacement vectors for the Lower Austrian mosaic are shown in Figure 2.

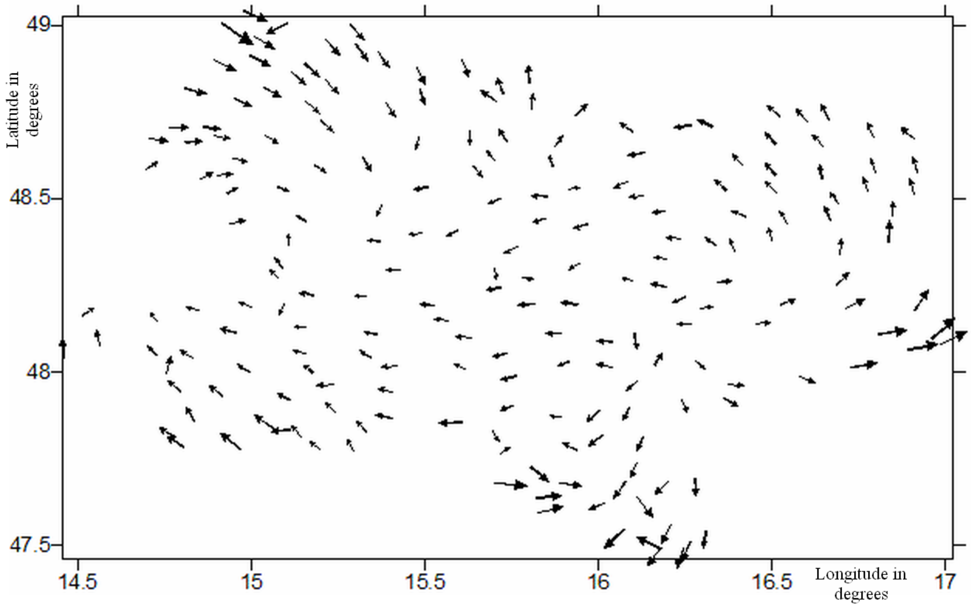

Figure 2

The remaining error vectors after using the best fitting Cassini-projections for the Lower Austrian mosaic, as a mathematical model of the horizontal control of the historical map mosaic

The longitude of the above-mentioned two regions, are quite interesting. In the case of Hungary, it is slightly east of Pest (a part of present-day Budapest), where we could not find any important landmark, either in modern maps or in the processed historical map sheet mosaic. The resulting longitude (19.29 degrees from Greenwich) runs within the mentioned error margin from the present-day geodetic observatory of Penc, which is a mere co-incidence.

The resulting central longitude for Lower Austria (16.35 degrees from Greenwich), however, interestingly enough, cuts into the downtown of the capital city, Vienna. It runs near the Schottentor at the western end of the historical city, just around 800 meters west of the most obvious landmark of the city, the Stephansdom.

In the case of Croatia the orientation of the map sheets is far from the normal 'north up' convention; the meridian convergence exceeds 10 degrees. Even this region can be geo-referred using the estimation of the longitude of projection origin. In this case, this theoretical central longitude does not cross the region but runs far (cca. 20 degrees) east of it. The central meridians of all investigated regions are shown in Table 1.

Table 1

Longitudes of the best fitting Cassini-projection origins in the investigated regions

Hungary	**Lower Austria**	**Bohemia**	**Croatia**	**Galicia**
19.29	16.35	18.79	37.36	39.19

5. DISCUSSION

The accuracy of the mosaic fitting to the modern coordinate systems has to be compared to the horizontal control of the previous georeferencing attempts of the First Military Survey sheets. These are published by Arcanum (2006), using the estimation of the 10 parameters of a simple quadratic polynomial connection between modern (UTM) coordinates and image pixel coordinates on virtual mosaics at identical GCPs. It is important to underline that the original map sheets could not be mapped in the UTM coordinate system, as it was defined almost 200 years later. The horizontal error of this quadratic fitting is of the very same order as that of the linear fitting to the best-fitting Cassini-projection. This is a good indication that the First Military Survey has a Cassini-projection.

The case of Lower Austria is even more convincing. According to the parameter estimation, the projection origin falls in the downtown of Vienna, and we can use the Stephansdom as a projection center with the same accuracy (with a small shift in the east-west location of the mosaic corner point). This is a useful contribution to the debate about the existence of geodetic network behind the First Military System.

6. CONCLUSIONS AND FURTHER REFINEMENT POSSIBILITIES

We suggest that the First Military Survey had a well-defined geodetic background, even if the connected geodetic point list has not yet been found in the archives. The Cassini-projection with the above-mentioned meridians provides an acceptable mathematical model for the geo-referencing of Hungary and Lower Austria (the investigated regions). The model reproduces the modern coordinates of the scanned pixels with an error of up to 2.4 and 3 kilometers.

A further correction step (which is not discussed here, as it does not concern the topic of inversion methods) can be compiled using local correction grids, based on the same input point list and the parameter estimation results [20] (Figure 3).

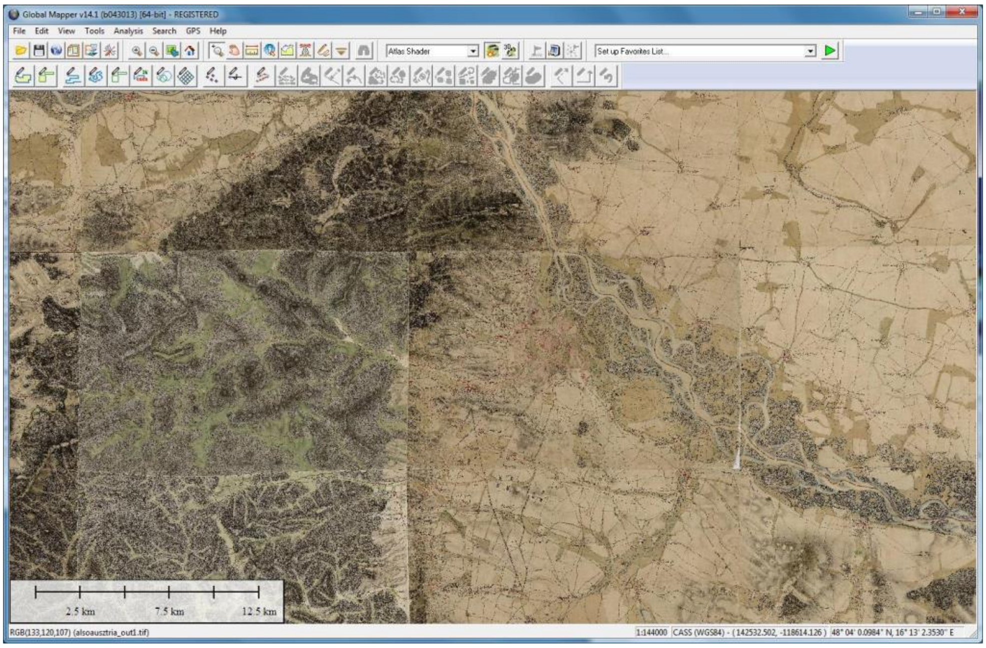

Figure 3

The originally linear boundaries of the rectangular map sheets are slightly distorted by the correction grid refinement in order to achieve better horizontal fit

7. List of Symbols

Symbol	Description	Unit
R	is the radius of Earth,	km
λ	longitude of the point	°
λ_0	longitude of the center of the projection	°
Φ	latitude of the point	°
Φ_0	latitude of the center of the projection	°
x, y	Cartesian coordinates of the projected point	m

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